

Supporting Information for "Fault-zone damage promotes pulse-like rupture and back-propagating fronts via quasi-static effects"

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Introduction Text S1 describes the methodology used in Section 3. Text S2 and S3 are mathematical derivations of results presented and discussed in the main text. Figures S1-S2 and S4 show supplementary aspects of the methodology used in Section 3. Figures S3 and S5 show supplementary results to those presented in the main text. Figure S6 is a summary of LVFZ properties contained in the literature. Table S1 describe supplementary

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information relative to our results in Section 3. Table S2 contains information required to reproduce our results in Section 3.

Text S1: Earthquake cycle simulations

Methods. We characterize the effect of a LVFZ on rise-time and slip profile in earthquake cycle simulations covering a wide range of values of LVFZ thickness and damage. We adopt a spectral boundary integral equation method (SBIEM) (Luo et al., 2017) and a quasi-dynamic approximation where wave-related effects are crudely represented by a radiation-damping term (Rice, 1993). The static stress transfer kernel required in the SBIEM for a LVFZ is derived in Text S2. The modeling approach captures the static LVFZ effects described in Section 2 of the main text, without including any dynamic effect of fault zone reflected waves, and its computational efficiency enables a comprehensive exploration of the parameter space.

The fault strength is prescribed by the Dieterich-Ruina rate-and-state friction law coupled with the “ageing law” of state evolution (Dieterich, 1981; Ruina, 1980, 1983):

$$\tau/\sigma = \mu_0 + a \ln \left(\frac{V}{V_0} \right) + b \ln \left(\frac{V_0 \theta}{D_c} \right) \quad (1)$$

$$\dot{\theta} = 1 - \frac{V\theta}{D_c} \quad (2)$$

where τ and σ are the shear and normal fault stresses, respectively, V is slip rate and θ a fault state variable. The parameter a quantifies the direct effect, b the evolution effect, and D_c is the characteristic slip related to the state evolution. Under steady-state, $b > a$ leads to stick-slip behavior (velocity-weakening, VW) whereas $b < a$ leads to stable sliding behavior (velocity-strengthening, VS). We represent a seismogenic zone driven by

surrounding creep by prescribing a VW patch of length L_{vw} in the middle of the fault, surrounded by two VS segments of total length $3L_{vw}$ and, at further distance, by steady uniform creep at slip rate V_{pl} (Fig. S2). The parameter values of our numerical model are given in Table S2.

Ruptures that start as a crack and later turn into a pulse require a minimum rupture distance to develop the transition, therefore a sufficiently large ratio between L_{vw} and the nucleation length L_{nuc} (Rubin & Ampuero, 2005):

$$L_{nuc} = \frac{2}{\pi} \frac{\mu D_c b}{\sigma (b - a)^2} \quad (3)$$

Previous earthquake cycle simulations including a LVFZ model did not show significant reductions in rise-time for $L_{vw} \sim 1.5L_{nuc}$ and $\Delta = 0.36$ (Kaneko et al., 2011), so we extended the seismogenic length to $L_{vw} \sim 10L_{nuc}$. Moreover, we explored values of Δ ranging from moderate damage to the upper bound inferred from current seismological observations ($\Delta = 0.5 - 0.9$) and values of h extending over a range wide enough to encompass most geological and seismological observations ($2h/L_{vw} \sim 10^{-3} - 10$). A compilation of observed or estimated levels of damage and fault-zone thickness in natural faults is given in Fig. S6.

We prescribed a minimum of five elements within an effective cohesive zone,

$$L_b^* = \frac{9\pi}{32} \frac{\mu^* D_c}{\sigma b}, \quad (4)$$

where μ^* is an effective shear modulus that accounts for the LVFZ derived below. A well-resolved cohesive zone was verified a posteriori in all our simulation results, characterized by a smooth and properly sampled stress distribution near the rupture tips.

Estimation of the process zone size in a LVFZ. The components of slip and stress drop at a wavenumber k^* are related by an effective shear modulus, μ^* , which represents an effective value of the shear modulus profile $\mu(x)$ up to an off-fault distance comparable to $\lambda^* = 2\pi/k^*$. As is the case of a homogeneous medium, the static stress transfer at this effective length scale can be written as:

$$\mathcal{K}(k^*) = \frac{1}{2}\mu^*|k^*| \quad (5)$$

Combining equations (17) and (5), we obtain:

$$\mu^* = \mu(1 - \Delta) \coth(h|k^*| + \operatorname{atanh}(1 - \Delta)) \quad (6)$$

An estimate of the process zone size in a LVFZ, L_b^* , is related to the process zone size in an intact medium, L_b , by:

$$L_b^* = L_b \frac{\mu^*}{\mu} \quad (7)$$

After replacing Eq. (6) into Eq. (7) and setting $\lambda^* \approx L_b^*$, the process zone in a LVFZ satisfies:

$$L_b^* = L_b(1 - \Delta) \coth\left(\frac{2\pi h}{L_b^*} + \operatorname{atanh}(1 - \Delta)\right) \quad (8)$$

This equation is solved numerically to obtain L_b^*/L_b as a function of Δ and h/L_b .

Text S2: The static stress transfer kernel in a LVFZ model.

Problem statement. Let us consider a 2D elastic medium where the fault is located on the line $x = 0$. The medium is heterogeneous with the shear modulus μ depending only on x . Slip is anti-plane in the direction out of the $x - y$ plane. A static slip distribution $\mathcal{D}(y)$ produces a shear stress on the fault $\mathcal{T}(y)$. Because the elasticity problem is linear,

slip and stress are related by a linear relation:

$$\mathcal{T}(y) = - \int_{-\infty}^{\infty} \mathcal{K}(y, y') \mathcal{D}(y') dy', \quad (9)$$

where \mathcal{K} is the static stress transfer kernel. The minus sign is introduced to give \mathcal{K} a meaning analogous to stiffness in a spring-block system. As the problem is invariant by translation along y ($\mathcal{K}(y, y') = \mathcal{K}(y - y')$), Eq. (9) is a convolution:

$$\mathcal{T}(y) = - \int_{-\infty}^{\infty} \mathcal{K}(y - y') \mathcal{D}(y') dy' \quad (10)$$

After a Fourier transform, the convolution simplifies into a product:

$$\mathcal{T}(k) = -\mathcal{K}(k) \mathcal{D}(k), \quad (11)$$

where k is the wavenumber along the fault direction, y . In order to connect stress and slip in the fault, the goal is to derive an expression for the static kernel in spectral domain, the so-called spectral stiffness $\mathcal{K}(k)$.

The equation of anti-plane elasticity governing the displacement field $u(x, y)$ parallel to z is:

$$\mu(x) u_{,yy} + (\mu(x) u_{,x})_{,x} = 0 \quad (12)$$

A first boundary condition is the symmetric distribution of applied slip on each side of the fault:

$$u(0^\pm, y) = \pm \frac{1}{2} \mathcal{D}(y) \quad (13)$$

A second boundary condition requires that displacement u must be finite at $x = \pm\infty$. As the boundary conditions are symmetric, the resulting displacements are symmetric as well and follow $u(-x, y) = -u(x, y)$. Therefore, we solve for the half-plane $x \geq 0$ only.

After applying a Fourier transform over Eq. (12), our task is reduced to find $u(x, k)$ such that:

$$\begin{aligned} -k^2 \mu(x)u + (\mu(x)u_{,x})_{,x} &= 0 \\ u(0^+, k) &= \frac{1}{2}\mathcal{D}(k) \\ u(\infty, k) &< 0 \end{aligned} \tag{14}$$

and then evaluate the fault shear stress $\mathcal{T}(k) = \mu(0)u(0, k)_{,x}$.

The problem has analytical solutions only for certain shear-modulus distributions $\mu(x)$. In the following we address two cases: a homogeneous medium and a two-layer medium. An analytical solution for an exponential distribution of $\mu(x)$ is possible as well but not exposed here (Ampuero et al., 2002).

Homogeneous medium. In a homogeneous medium, Eq. (12) reduces to:

$$-k^2 u + u_{,xx} = 0, \tag{15}$$

Its well-known general solution is $u(x, k) = A \exp(-|k|x) + B \exp(|k|x)$. The finite displacement boundary condition imposes $B = 0$, and the fault boundary condition at $x = 0$ implies $A = \frac{1}{2}\mathcal{D}$. The resulting displacement is $u(x, k) = \frac{1}{2}\mathcal{D}(k) \exp(-|k|x)$. After evaluating the shear stress on the fault, $\mathcal{T}(k) = -\frac{1}{2}\mu|k|\mathcal{D}(k)$, the spectral stiffness is:

$$\mathcal{K}(k) = -\mathcal{T}(k)/\mathcal{D}(k) = \frac{1}{2}\mu|k| \tag{16}$$

Two-layer medium. Consider a fault in a homogeneous medium surrounded by two layers of uniform half-thickness h and homogeneous but reduced shear modulus (Fig. 1b). Within the layers the shear modulus is $\mu(1 - \Delta)$ and outside the layers it is μ . The deriva-

tion of the kernel in a layered medium follows the same steps as the previously addressed case. The differential equation is Eq. (15). Its general solution is a combination of exponential functions for each layer, together with a total of four new constants analogous to A and B . Two new boundary conditions arise: continuity of displacement and stress across the interface between the layers, $u(h^+, k) = u(h^-, k)$ and $(1 - \Delta)u(h^-, k)_{,x} = u(h^+, k)_{,x}$.

It is possible to obtain the displacement $\mathcal{D}(x, k)$ after some algebraic work, then derive the shear stress at the fault and finally the spectral kernel:

$$\mathcal{K}(k) = \frac{1}{2}\mu|k|(1 - \Delta) \coth(h|k| + \operatorname{atanh}(1 - \Delta)) \quad (17)$$

Static slip profiles with constant stress drop. A first application of Eq. (17) is to numerically compute the static slip profiles of a rupture with prescribed constant stress drop propagating in a fault with a LVFZ (Fig. 1c). By applying an inverse fast Fourier transform to Eq. (17) over a very long fault, we obtained a static stress transfer kernel in space domain, $\mathcal{K}(y)$. Then assuming a uniform stress drop, we solved numerically the discretized version of Eq. (10) to obtain the slip profiles.

Numerical implementation of a LVFZ. Our numerical implementation of a LVFZ on multi-cycle earthquake simulations consists of combining the time-domain kernel of a fault with finite length (Cochard & Rice, 1997) with Eq. (17) in the frequency domain. The numerical models shown in Fig. 3 are based on this implementation. We verified that the values of the obtained kernel are similar to those obtained by the more expensive approach of applying Eq. (17) to a periodic homogeneous fault 32 times longer (Fig. S4).

Text S3: The stress transfer in a LVFZ and a Burridge-Knopoff model.

Burridge-Knopoff (BK) model. In a BK model (Burridge & Knopoff, 1967), the quasi-static slip \mathcal{D}_i and stress \mathcal{T}_i at the base of the i -th block of area dy^2 relate to each other as:

$$\mathcal{T}_i dy^2 = -k_L(\mathcal{D}_i - \mathcal{D}_L) + \bar{\mathcal{K}}(\mathcal{D}_{i-1} - 2\mathcal{D}_i + \mathcal{D}_{i+1}), \quad (18)$$

where \mathcal{D}_L is the loading displacement, k_L the loading stiffness, and $\bar{\mathcal{K}}$ the stress transfer due to the relative motion of the sliders. We furthermore introduce a loading stiffness per unit area of block surface defined as:

$$\bar{\mathcal{K}}_L = k_L/dy^2 \quad (19)$$

Taking the continuum limit ($dy \rightarrow 0$) in Eq. (18):

$$\mathcal{T}(y) = -\bar{\mathcal{K}}_L(\mathcal{D}(y) - \mathcal{D}_L) + \bar{\mathcal{K}}\mathcal{D}_{,yy} \quad (20)$$

The second term in the right-hand side in the equation above is derived by expanding terms in a Taylor series up to second order at small dy :

$$\mathcal{D}_{,yy} \approx \frac{\mathcal{D}(y - dy) - 2\mathcal{D}(y) + \mathcal{D}(y + dy)}{dy^2} = \frac{\mathcal{D}_{i-1} - 2\mathcal{D}_i + \mathcal{D}_{i+1}}{dy^2} \quad (21)$$

Taking the Fourier transform for non-zero wavenumbers ($|k| > 0$):

$$\mathcal{T}(k) = -\bar{\mathcal{K}}_L\mathcal{D}(k) - \bar{\mathcal{K}}k^2\mathcal{D}(k) = -(\bar{\mathcal{K}}_L + \bar{\mathcal{K}}k^2)\mathcal{D}(k) \quad (22)$$

The loading displacement \mathcal{D}_L is spatially uniform, hence it only contributes when $k = 0$.

We get the following static kernel in spectral domain:

$$\mathcal{K}(k) = \bar{\mathcal{K}}_L + \bar{\mathcal{K}}k^2 \quad (23)$$

Static slip induced by uniform stress drop in the continuum BK model.

Consider a uniform stress drop within a rupture segment of size r , i.e.

$$\Delta\tau = \bar{\mathcal{K}}_L \mathcal{D}(y) - \bar{\mathcal{K}} \mathcal{D}_{,yy} \quad (24)$$

for $y \in [-r/2, r/2]$, and zero slip elsewhere. The solution to the second-order linear ODE above is the sum of a particular solution satisfying the ODE (here, a uniform slip profile) and the general solution to the homogeneous version of the ODE:

$$\mathcal{D}(y) = \frac{\Delta\tau}{\bar{\mathcal{K}}_L} + Ae^{\kappa y} + Be^{-\kappa y} \quad (25)$$

where $\kappa = \sqrt{\bar{\mathcal{K}}_L/\bar{\mathcal{K}}}$. The constants A and B are determined by enforcing the boundary conditions at the rupture tips, $\mathcal{D}(\pm r/2) = 0$:

$$A = B = -\frac{\Delta\tau}{2\bar{\mathcal{K}}_L \cosh(\kappa r/2)} \quad (26)$$

Thus, the slip profile is

$$\mathcal{D}(y) = \frac{\Delta\tau}{\bar{\mathcal{K}}_L} \left(1 - \frac{\cosh(\kappa y)}{\cosh(\kappa r/2)} \right) \quad (27)$$

Figure S7 shows the resulting slip profiles for a range of values of the dimensionless number κr . For large κr values, the slip is flat over most of the rupture, as in pulse-like ruptures, with slip approximately equal to $\Delta\tau/\bar{\mathcal{K}}_L$.

Comparison between LVFZ and BK kernels. For a LVFZ model, we rewrite the kernel given in Eq. (17) as:

$$\mathcal{K}(k) = \frac{1}{2}\mu_d|k| \left(\frac{1 + (1 - \Delta) \tanh(1 - \Delta)}{1 - \Delta + \tanh(h|k|)} \right) \quad (28)$$

In a highly damaged fault zone where $\Delta \rightarrow 1$, the kernel reduces to:

$$\mathcal{K}(k) \approx \frac{1}{2}\mu_d|k| \left(\frac{1}{1 - \Delta + \tanh(h|k|)} \right) \quad (29)$$

The high-frequency regime is defined by $\tanh(h|k|) \gg 1 - \Delta$ and leads to:

$$\mathcal{K}(k) \approx \frac{1}{2}\mu_d|k| \coth(h|k|) \quad (30)$$

Moreover, if $h|k| \ll 1$, by Taylor expansion we obtain:

$$\mathcal{K}(k) \approx \frac{\mu_d}{2h} + \frac{1}{6}\mu_d h k^2 \quad (31)$$

This shows that, under certain conditions for $h|k|$, the stress transfer of the LVFZ model is equivalent to that of the BK model, with the following formal analogies:

$$\bar{\mathcal{K}}_L = \frac{\mu_d}{2h} \quad (32)$$

$$\bar{\mathcal{K}} = \frac{\mu_d h}{6} \quad (33)$$

Figure S8a shows the LVFZ kernels (Eq. 17) for various damage levels and their BK-like approximation (Eq. 31). Figure S8 shows their ratio as a function of normalized wavenumber kh and for all damage levels Δ above 0.5. The bandwidth over which the two kernels agree expands with increasing damage.

Under the conditions described above, the similarity between the LVFZ and BK kernels implies a formal analogy between the two models, which we now exploit to develop implications on pulse-like rupture. Eqs. (32) and (33) give $\kappa = \sqrt{3}/h$. The condition $\kappa r \gg 1$ for a flat, pulse-like slip profile in the BK model becomes, for the LVFZ model, $r \gg h/\sqrt{3}$. Under that condition and assuming a uniform stress drop $\Delta\tau$, a LVFZ produces ruptures

with the flat slip profile characteristic of pulses and average slip of

$$\mathcal{D} \approx \frac{\Delta\tau}{\bar{\mathcal{K}}_L} = \frac{2h\Delta\tau}{(1-\Delta)\mu} \quad (34)$$

In the low-frequency regime defined by $\tanh(h|k|) \ll 1 - \Delta$, the LVFZ is too narrow to have an effect on the stress transfer, and the kernel tends to that of a homogeneous medium:

$$\mathcal{K}(k) \approx \frac{1}{2}\mu|k| \quad (35)$$

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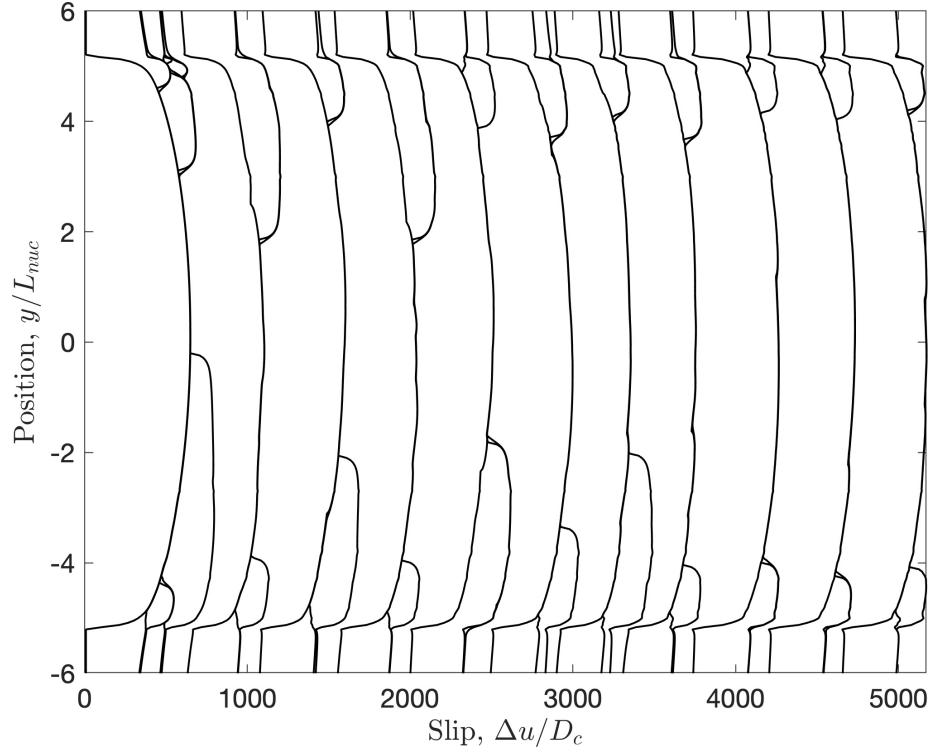


Figure S1. History of seismic activity in a simulation of a fault model with $\Delta = 0.9$ and $2h \approx 1/40L_{vw}$. Solid lines represent accumulated slip after an earthquake occurs.

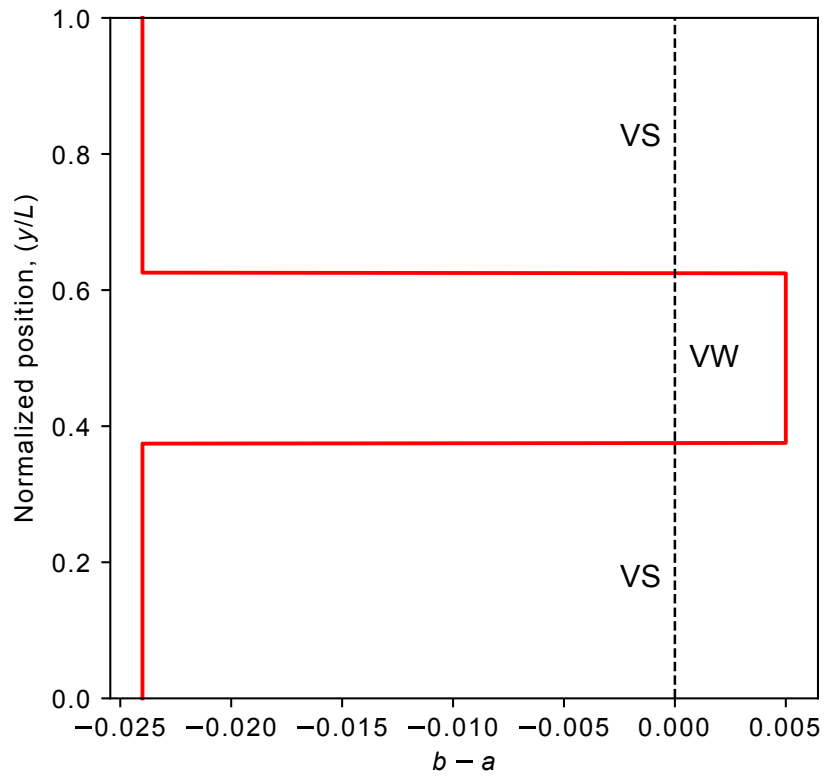


Figure S2. The $b-a$ parameter along position in the fault. The seismogenic zone is indicated as a velocity-weakening zone (VW) surrounded by two stably sliding velocity-strengthening (VS) segments.

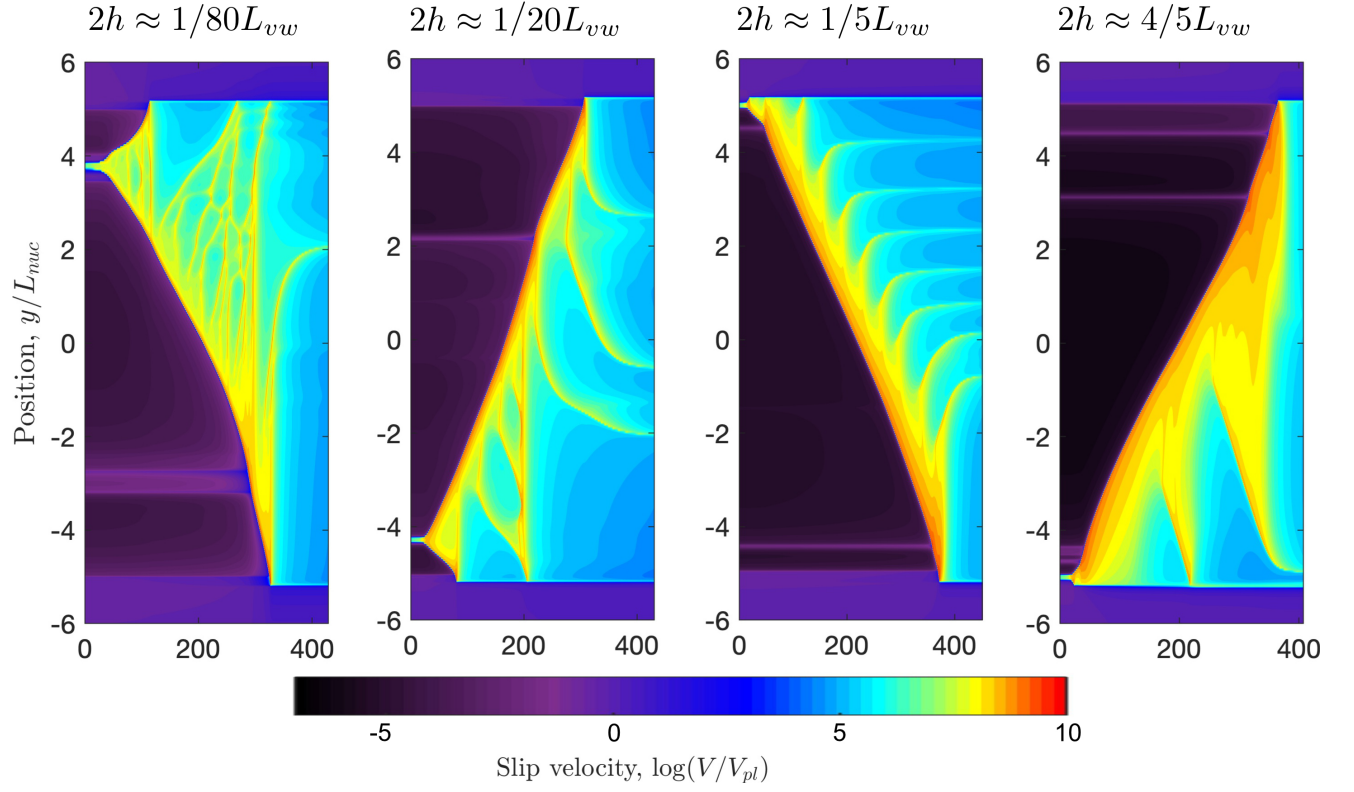


Figure S3. Spatiotemporal evolution of slip rate in the characteristic event of earthquake cycle models using a LVFZ with $\Delta = 0.9$ and different values of damage zone thickness.

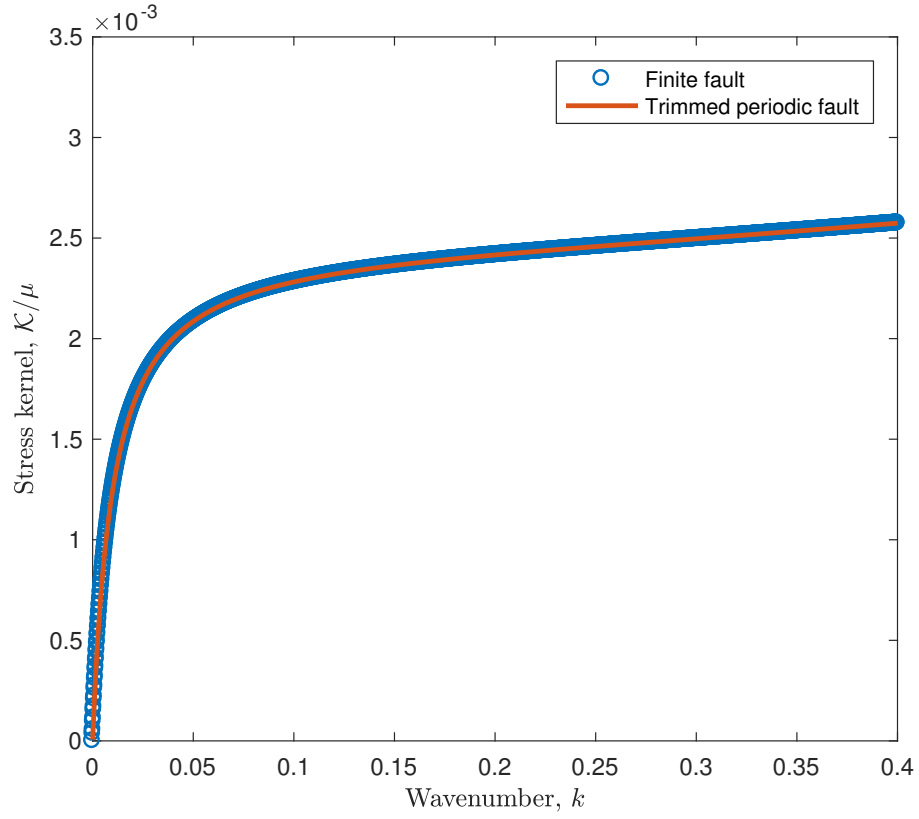


Figure S4. Static stress kernel in a LVFZ versus wavenumber based on two numerical implementations. The blue circles represent the combination of the kernel of a finite fault with Eq. (17) in the wavenumber domain. The continuous orange line is an approximated kernel using Eq. (17) over a periodic fault 32 times longer.

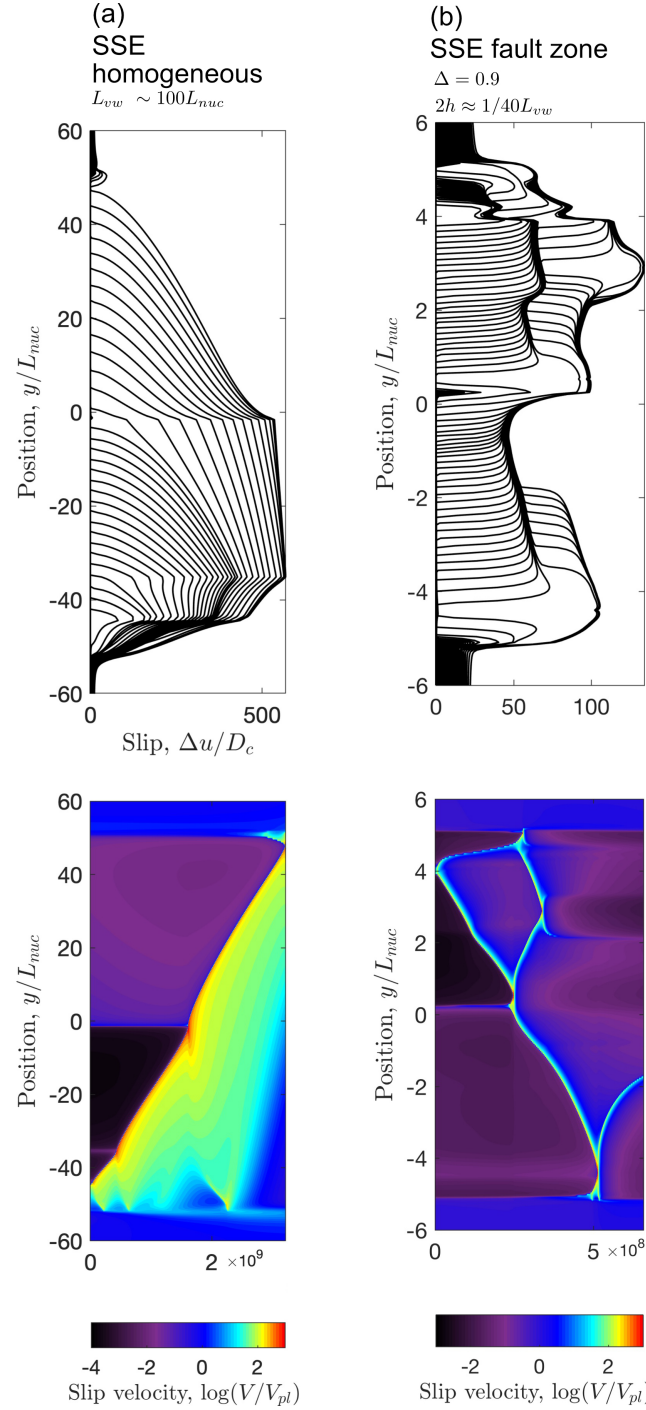


Figure S5. Comparison of slow-slip earthquakes (SSE) with and without a fault zone (LVFZ) modeled using a friction law of increased velocity-strengthening. (a) A slow earthquake in a fully-damaged homogeneous medium. The slip profiles are indicative of crack-like rupture propagation. (b) The addition of the LVFZ promotes multiple back-propagating secondary fronts. The main and secondary fronts show slip profiles indicative of pulse-like rupture propagation.

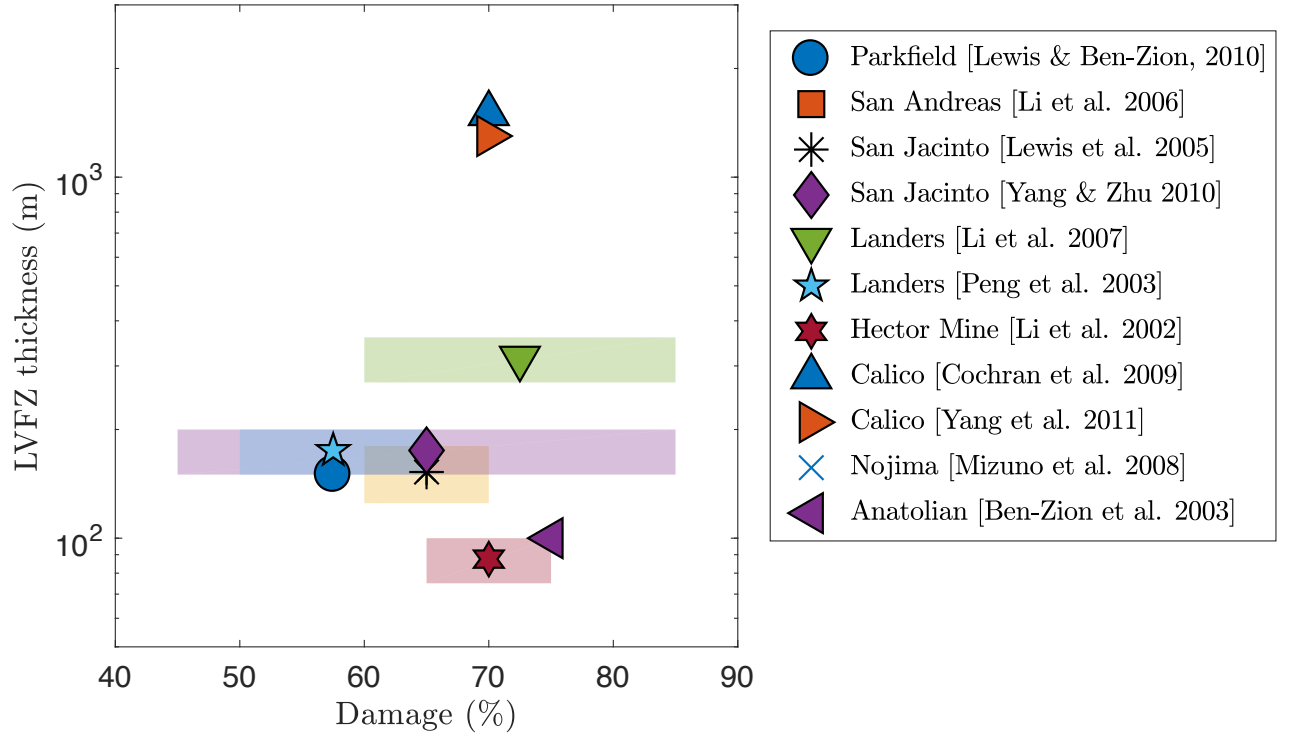


Figure S6. A compilation of fault-zone properties included in Huang et al. (2014). Damage represents a reduction in shear moduli, which relates to a reduction in velocity as $\Delta_\mu = 1 - (1 - \Delta_v)^2$, where Δ_μ is the damage reported in the figure and Δ_v the reduction in velocity reported in Huang et al. (2014).

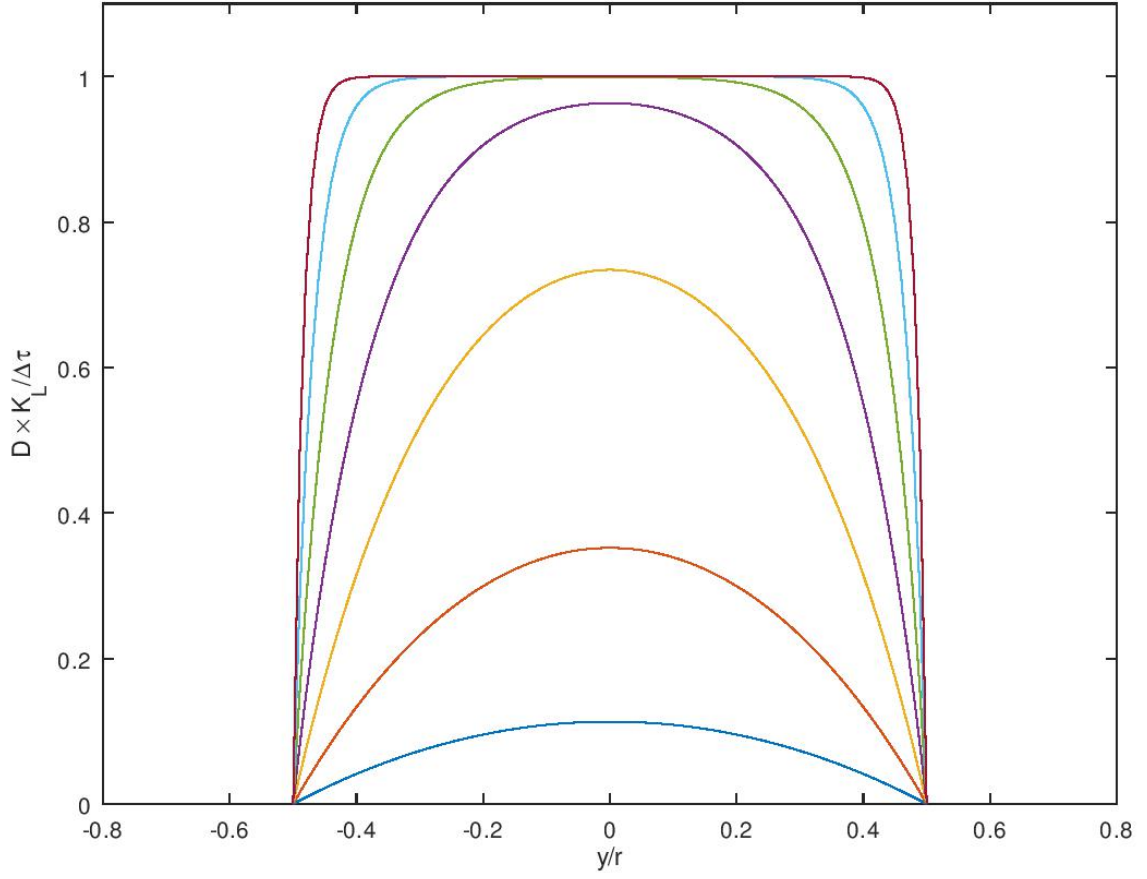


Figure S7. Static slip profiles for ruptures with uniform stress drop $\Delta\tau$ in a continuum Burridge-Knopoff model, with loading stiffness K_L . Each curve is for a different value of κr in $\{1, 2, 4, 8, 16, 32, 64\}$, from bottom to top.

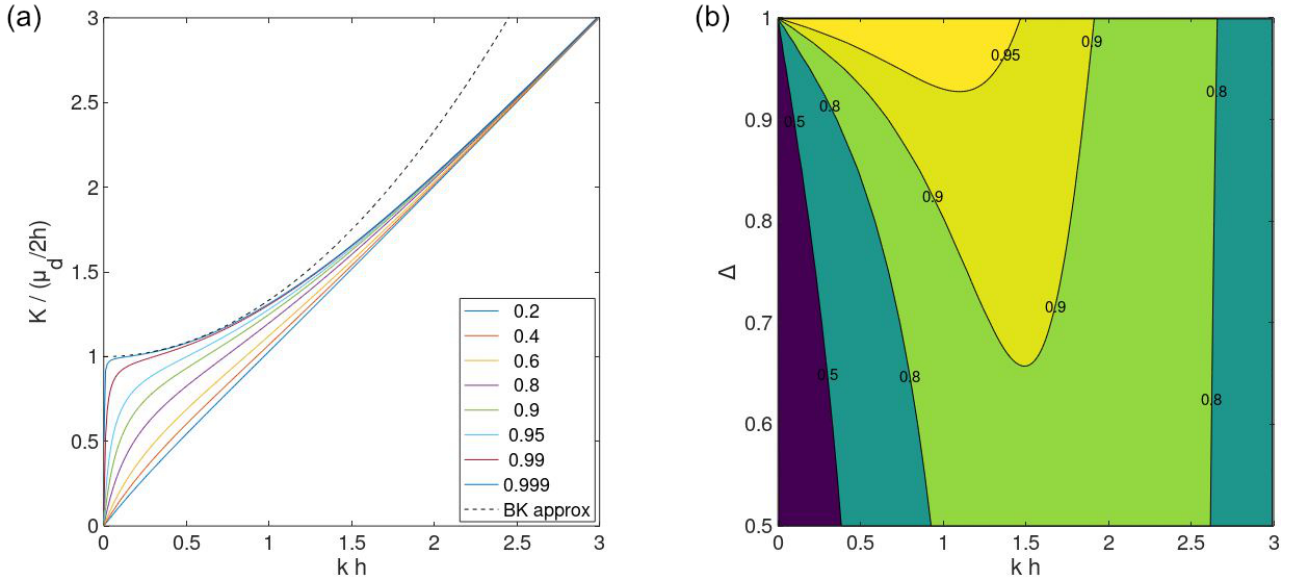


Figure S8. Comparison of LVFZ and BK static stress transfer kernels. (a) LVFZ kernels for a range of damage levels Δ (see legend) and their BK approximation (dashed) as a function of normalized wavenumber kh . Kernels are normalized by $\mu_d/2h$. (b) Ratio between LVFZ kernels and their BK-like approximation, as a function of kh and damage level Δ . The bandwidth over which the two kernels agree expands with increasing damage (see for instance the contours 0.9 and 0.95).

Table S1. Approximated dimensions of a fault and properties of the rupture in the homogeneous and LVFZ models shown in Fig. 3 after assuming a value for D_c .

	Homogeneous	LVFZ ($\Delta = 0.9$)
Characteristic slip, D_c	2 mm	2 mm
Seismogenic zone, L_{vw}	2.5 km	2.5 km
Nucleation length, L_{nuc} and L_{nuc}^*	242 m	40 m
Fault-zone thickness, $2h$	-	64 m
Event duration, t	2.4 sec	32.4 sec
Average rise-time	2.1 sec	5.6 sec

Table S2. Parameters used in the numerical simulations.

Parameter	Symbol	Value
Direct effect (VW/VS)	a	0.014/0.043
Evolution effect (VW/VS)	b	0.019/0.019
Characteristic slip	D_c	2 mm
Tectonic loading	V_{pl}	10^{-9} m/s
Reference slip rate	V_0	10^{-9} m/s
Reference friction coefficient	μ_0	0.6
Shear-wave speed	β	3.5 km/s
Intact shear modulus	μ	30 GPa
Effective normal stress	σ	120 MPa